Second Midterm Examination

Mathematics 167, Game Theory

Ferguson — Fri. May 20, 2005

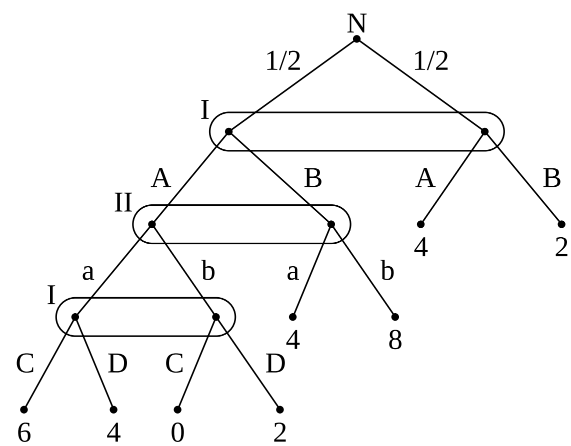
1. Player II chooses one of the numbers in the set $\{1, 2, 3\}$. Then one of the numbers not chosen is selected at random and shown to Player I. Then Player I guesses which number Player II chose, winning that number if he is correct and winning nothing otherwise.

(a) Draw the Kuhn Tree.

(b) Describe Player I's behavioral strategies for this game.

2. (a) Given a game with the following Kuhn tree, find the equivalent strategic form of the game.

(b) Solve.



3. Consider the non-zero-sum game with bimatrix, $\begin{pmatrix} (2,1) & (5,2) & (3,1) \\ (3,4) & (4,3) & (3,0) \end{pmatrix}$.

(a) Find the safety levels of the players.

(b) Find the maxmin strategies of the players.

4. Consider the bimatrix game: $\begin{pmatrix} (4,3) & (5,2) & (5,3) \\ (5,3) & (3,4) & (5,2) \\ (2,4) & (4,1) & (5,4) \end{pmatrix}$

(a) Find all PSE's.

(b) Remove I's (weakly) dominated row and remove II's (weakly) dominated column. In the resulting 2 by 2 game, find the SE given by the equalizing strategies.

5. In the Stackelberg Model, the players may have different production costs. Suppose Player I's production cost is 2 per unit, and Player II's production cost is 1 per unit. (There is no setup cost.) Player I, the "leader", announces the amount, q_1, he will produce, and then Player II chooses an amount, q_2, to produce. The price function is $P(q_1, q_2) = (19 - q_1 - q_2)^+$.

(a) How much should Player II produce, as a function of q_1?

(b) How much should Player I produce?

(c) How much, then, does Player II produce?

Solution to the Second Midterm Examination
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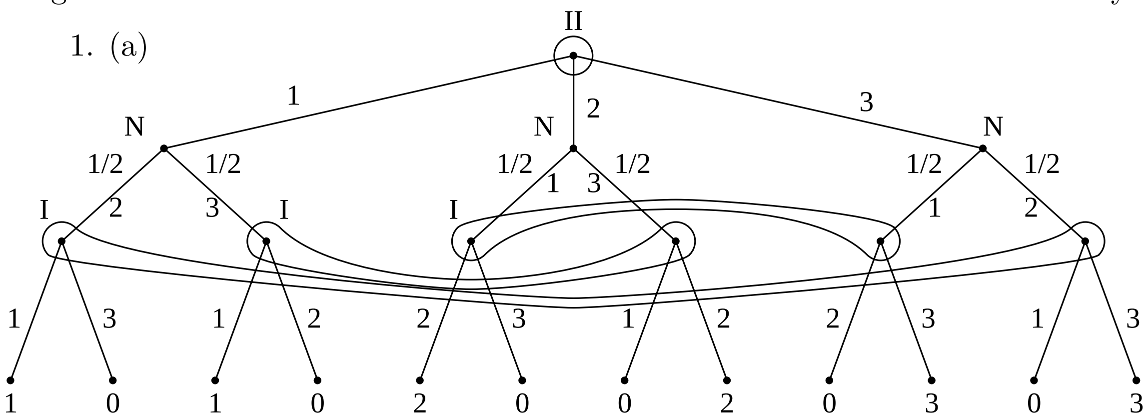
1. (a)



(b) A behavioral strategy for Player I specifies how he will randomize in each of his information sets. Let p_1 be the probability of guessing 2 if he hears 1 (so $1-p_1$ is the probability of guessing 3 if he hears 1), let p_2 be the probability he guesses 3 if he hears 2, and let p_3 be the probability he guesses 1 if he hears 3. A behavioral strategy for I is then the triplet (p_1, p_2, p_3).

2. (a) $\begin{array}{c} \\ AC \\ AD \\ BC \\ BD \end{array} \begin{array}{c} \quad a \quad b \\ \begin{pmatrix} 5 & 2 \\ 4 & 3 \\ 3 & 5 \\ 3 & 5 \end{pmatrix} \end{array}$. (b) The value is $V = 19/5$, $p = (2/5, 0, 3/5, 0)$ is optimal for I, and $q = (3/5, 2/5)$ is optimal for II.

3. (a) There is a saddle point in Player I's matrix at $\langle 2, 1\rangle$, with value $v_I = 3$. There is a saddle point in (the transpose of) Player II's matrix at $\langle 1, 2\rangle$, with value $v_{II} = 2$.

(b) I's maxmin strategy is row 2. I's maxmin strategy is column 2.

4. (a) There are PSE's at (row 1, col 3) and (row 3,col 3).

(b) Row 3 is dominated by row 1. Col 3 is dominated by col 1. On the resulting 2 by 2 bimatrix, (1/2,1/2) by I is equalizing on II's matrix, and (2/3,1/3) by II is equalizing on I's matrix. So $((1/2, 1/2, 0), (2/3, 1/3, 0))$ is an SE.

5. (a) $u_2(q_1, q_2) = q_2(19 - q_1 - q_2) - q_2$, and $\partial u_2/\partial q_2 = -2q_2 + 19 - q_1 - 1$. Setting to zero and solving gives $q_2 = (18 - q_1)/2$.

(b) $u_1(q_1, q_2(q_1)) = q_1(19 - q_1 - (18 - q_1)/2) - 2q_1$, and $\partial u_1/\partial q_1 = -q_1 + 8$. So $q_1 = 8$ is the optimal production.

(c) Therefore, $q_2 = (18 - 8)/2 = 5$.